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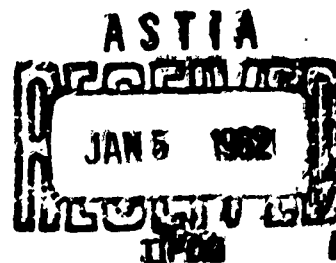
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# 1. MOVING STRIATIONS IN GLOW DISCHARGE PLASMAS

H. S. Robertson and M. A. Hakeem

Supported by A. E. C. Contract No. AT-(40-1) - 2454

## A. Introduction

Oscillations and striations are quite common and well-known properties of direct current glow and arc discharges. While the positive column of such a discharge is a most convenient plasma, its inhomogeneities and fluctuations often preclude its use in the study of plasmas per se. Although it is fairly clear that moving striations are not closely related to the various plasma waves and instabilities predicted by theory, it is nevertheless of fundamental importance that they be clearly understood.

Recent theories of moving striations<sup>1), 2), 3)</sup>, as distinct from those of plasma oscillations, attribute the striation phenomenon to space charge waves associated with production and loss of ions. Watanabe and Oleson<sup>2)</sup> base their calculation on a constant ionization rate per electron and obtain simplified expressions for wave length and frequency which depend upon the magnitude of the electric field in the plasma. Their results do not agree well with experiment but they do demonstrate that ionization waves may well exist in a plasma. Pekarek and Krejčí,<sup>4)</sup> in very recent work, have shown in an over simplified but instructive calculation that ionization waves may exist if  $\frac{\partial \alpha}{\partial E} > 0$  ,

where  $Z$  is the ionization rate per electron and  $E$  is the magnitude of the electric field in the plasma.

A more complex theory by Robertson<sup>1)</sup>, in which expressions for the ionization rate satisfy the criterion that  $\frac{\partial Z}{\partial E} > 0$ , indicates that axial diffusion and ion losses to the walls would inhibit or suppress entirely any ionization waves which might be generated in the plasma, unless metastable atoms were present. Even more strongly, the quantity  $M \frac{\partial F}{\partial N}$ , where  $M$  is the metastable concentration,  $F$  is the total rate of ionization of the metastables by electrons and  $N$  is the electron concentration, must be positive and large enough to offset the stabilizing effects of diffusion and wall losses in order for moving striations to be present. This prediction is subject to experimental verification, some of which will be reported in this paper.

#### B. The Nature of Moving Striations

In an incompletely ionized gas, such as the plasma of a typical glow discharge, random fluctuations in local ion and electron densities lead to fluctuations in the electric field, and vice versa. If ionization is disregarded, the usual plasma oscillations may be derived from a theoretical analysis of the interaction of fluctuating fields and particle densities. When ionization is considered, and when the ionization rate is dependent upon both the magnitude of the electric field and the electron density, both of which may fluctuate but not independently, a

much more complex chain of events takes place.

A local diminution of the electron density leads to a local increase in the electric field, with consequent enhancement of the ionization rate per electron. Positive ion density fluctuations could also lead to increased local electric fields and ionization rates. The higher ionization rate may, by locally producing more charge carriers than are required for steady state conduction, cause either local oscillation in the production rate or, more plausibly, a wave of ionization to propagate along the plasma in the direction of the electric field. Electron and ion diffusion, both axially and radially, and recombination, all act to smooth out the fluctuations and make the plasma more stable.

Another important stabilizing effect is that apparently <sup>1)</sup> the total direct ionization rate, not just the ionization rate per electron, decreases as the electron density increases at constant current density. The ionization rate per electron increases rapidly with electric field in the usual range of positive column electric fields. The field, at constant current density, is approximately inversely proportional to electron density, and ionization rate per electron turns out to be, for the usual range of conditions in the positive column, a monotone decreasing function of the electron density. This effect contributes very strongly to the stability of the positive column.

When metastable atoms are present in quantity however, the

ionization rate may for a time continue to increase as the electron population increases. This is because, in the inert gases for example, it is much easier to ionize a metastable than an unexcited atom, and a decrease in the local electric field has a less drastic effect on the rate per electron for ionizing metastables than on that for ionizing unexcited atoms. Thus the presence of metastables can lead to the growth, rather than the suppression, of instabilities. On this basis it was predicted<sup>1)</sup> that moving striations are critically dependent upon the metastable density and the electron density, or the electric field, in the positive column.

The predicted dependence upon metastables is not worked out in correct quantitative form, since the equations governing growth, loss, and flow of particles in the positive column are quite complicated, even in an approximate form are non-linear, and depend upon electron velocity distribution, which is also variable and not known. A really correct solution to the problem, based on microphysical collision processes, seems to be still beyond the realm of possibility. For this reason, three critical experiments were carried out to check the validity of the prediction.

Moving striations could result from instabilities in the plasma per se, or from instabilities or relaxation oscillations originating at a boundary and propagating through the plasma. Both kinds of striations are observed experimentally, as will be discussed later.



In any case, the qualitative arguments about stability indicate that unless the metastable mechanism is operative, instabilities and local perturbations should be attenuated as they propagate from their origin, and moving striations should not be observed.

### C. Experimental Results

#### C.1 Alkali Vapor Plasmas

An obvious experimental check on the necessity of metastables for the existence of moving striations is to examine positive column plasmas which contain no metastables. The alkali metals are the most likely choices, since there are no metastable states of neutral alkali atoms. Consequently, observations were made on plasmas in K, Rb, and Cs, over the current range 0-400 ma, in tubes from 1 cm to 3 cm i.d. and 20 - 30 cm between electrodes, at vapor pressures from 0.01 to 2.0 mm Hg. <sup>5)</sup> . Photomultipliers, Langmuir probes, and oscillographic observation of tube voltage have been used as means of detecting moving striations. At no time have moving striations been observed.

Two other pertinent observations have been made on alkali vapor plasmas. First, there is frequently an oscillation at the anode, associated with one or more anode spots. This causes a tube voltage oscillation of about one volt, with a period dependent upon current, pressure, material and anode geometry. These oscillations are usually periodic and can be seen clearly by a phototube at the anode.

The oscillograms of tube voltage are like those seen when moving striations are present. The phototube shows that the perturbation is propagated from the anode toward the cathode, but attenuated rapidly. At distances greater than 1 cm. from the anode, no trace of the oscillation may be seen in the emitted light.

Second, when multiple spots are present on the anode, the voltage oscillogram often shows a very complicated, but periodic, behavior. This has been found to result from the anode spots interacting with each other and triggering, in sequence, the individual relaxation oscillations. In a responsive plasma, this sort of activity at the anode could produce the complex periodicities often observed in moving striations. Indeed this is just the effect reported by Cooper <sup>6)</sup> in argon. Zaitsev <sup>7)</sup> also reports striations which originate from oscillations in the anode region.

A detailed report of our work on anode spot oscillations is in preparation.

Oscillographic measurements made with floating probes indicate that the velocity of propagation of the signal produced by the anode spot along the plasma is in excess of  $10^6$  m/sec. which is much faster than striation velocities. Externally generated signals, applied to a floating probe, are propagated to the anode and to other probes also very rapidly. The behavior seems to be nothing more than that of a good conductor in contact with an oscillator at one point carrying the

signal to other points. No modulation of the light in the plasma is observed as a consequence of externally generated oscillations applied to the probes.

In magnetic fields, violent oscillations and a variety of fluctuation phenomena are observed which have little to do with moving striations. For example, the rotating luminous helices observed by Allen, Paulikas, and Pyle<sup>8)</sup>, which account for the observations of Lehnert<sup>9)</sup>, are clearly and easily produced in alkali plasmas, and are easier to study because of the absence of moving striations.

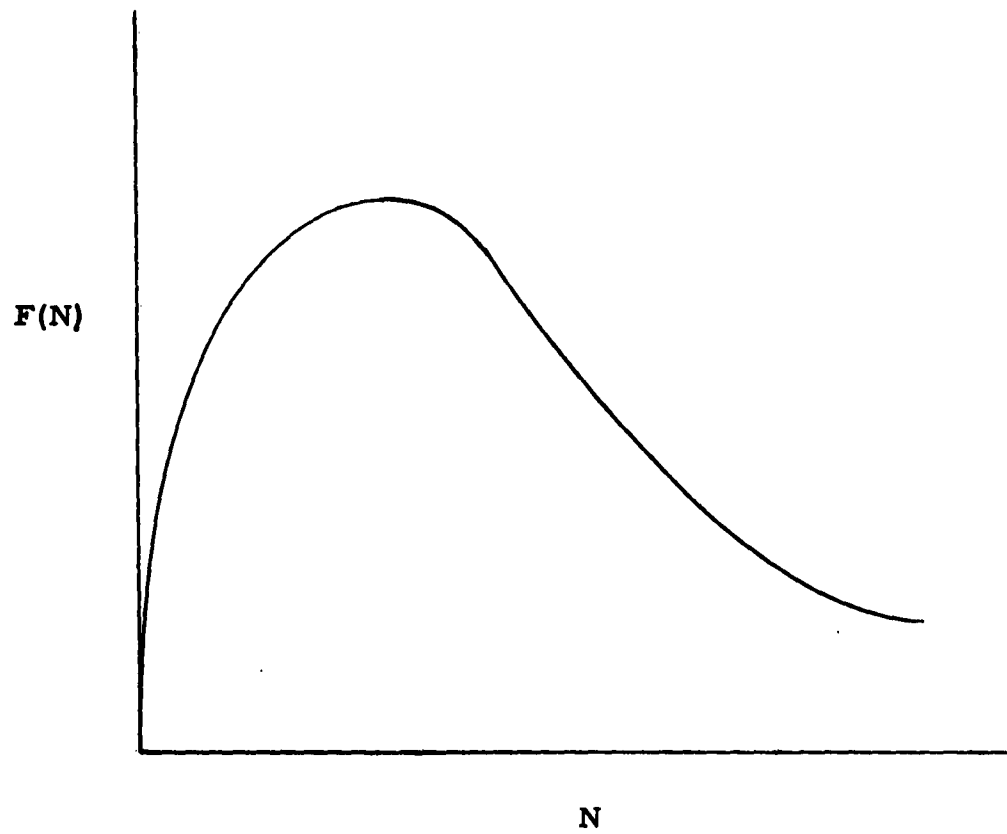
## C.2 Inert Gas Experiments

A second way to check on the role of metastables in the production of moving striations is to alter the metastable population and observe changes in the striation behavior. Meissner and Miller<sup>10)</sup> have clearly demonstrated that the metastable population in inert gases can be high enough to have considerable influence on the plasma characteristics. An inert gas discharge was irradiated strongly by them with light from another tube containing the same gas. Metastables in the irradiated tube absorb incident radiation in resonance, and become excited to levels from which radiative transitions to the ground state (or to non-metastable intermediate states) are possible. Thus they depopulated the metastable levels. The tube voltage for a particular current is perhaps tens of volts higher during irradiation than without it.

We have used a similar technique to observe the behavior of moving striations. The experimental tube is 30 cm. between tantalum cup electrodes and 1.2 cm inside diameter. Two somewhat longer tubes, 2.6 cm. i.d., are placed close to the experimental tube for sources of radiation. A black paper light shield can be used to screen the experimental tube from incident light without interfering with possible electrical coupling effects. Oscillographic inspection of the tube voltage of the experimental tube was used as the simplest indicator of moving striation behavior.

Experiments carried out in neon confirm Meissner and Miller's results and show, in some ranges of pressure and current, an extremely sensitive dependence of striations on the metastable population.

Assessment of the role of metastables by observing the effects of irradiative depopulation is not as simple as might be expected, since reduction of the metastable density produces consequent reduction in the ionization rate and thereby the electron density. The electric field therefore increases, and the ionization rate term  $MF(N)$  is changed by the reduction of both  $M$  and  $N$ . The form of  $F(N)$ , as estimated in ref. 1), is shown in Fig. 1. Moving striations are expected when  $M \frac{dF}{dN}$  is sufficiently positive. In the low current region,  $N$  is low, and irradiation can actually increase the product  $M \frac{dF}{dN}$



**Fig. 1,  $F(N)$ , where  $MF(N)$  is the ionization rate per unit volume of metastable atoms,  $M$  is the metastable concentration, and  $N$  is the electron concentration.**

while reducing  $M$ . At somewhat higher currents, the value of  $\frac{dF}{dN}$  is lower since  $N$  is higher. In this range, the product decreases as  $M$  decreases. At still higher currents,  $N$  is so large that  $\frac{dF}{dN}$  is negative, and striations should not be found.

Experimentally, observations are in complete qualitative agreement with these predictions. For example, with the illuminating tubes carrying 250 ma at 3.2 mm Hg, and the experimental tube at 0.83 mm Hg both filled with very pure Ne, at 5 ma moving striations were absent in the experimental tube unless it was illuminated. With illumination, the period of the tube voltage oscillation was not commensurate with that of the illuminating tube. At 10 ma, on the other hand, oscillations and moving striations were present in the experimental tube with no illumination, but disappeared completely when the light shield was removed.

When the experimental tube was shielded from light except for a region 4.5 cm long at the cathode end of the positive column, striking changes could be made in the experimental tube striation pattern by very small changes in the illumination level. For example, the additional illumination reflected from a sheet of white paper held near the tubes was sometimes sufficient to reduce the amplitude of the tube voltage oscillation from several volts to zero. Removal of the paper allowed the oscillation to return immediately.

At 0.45 mm Hg and 30 ma in the experimental tube, no oscilla-

tions were present. Faint illumination periodically produced transient damped oscillations in the tube voltage with the period between successive appearances equal to that of the voltage oscillation of the illuminating tube. The damped oscillations resembled those reported by Pekárek<sup>11)</sup> as resulting from applied voltage pulses. Here they are apparently produced by the periodic metastable depopulation in the experimental tube as a consequence of moving striations in the irradiating tubes. At higher levels of illumination the voltage oscillation in the experimental tube becomes much greater in amplitude, and follows accurately the periodic forcing of the irradiating tubes. This is expected, since a periodic irradiative depopulation of metastables in the experimental tube should certainly cause periodic changes in conductivity.

It should be mentioned that in neon, as well as in other inert gases observed by us previously, moving striations are clearly originated in the plasma itself sometimes, and in connection with anode spot oscillations under different conditions. Also at times both mechanisms are operative, sometimes independently and at other times apparently interacting strongly enough to exhibit the same periodicity. Cooper<sup>6)</sup> has reported experiments in tubes with abrupt changes in radius which exhibit moving striations of incommensurate periods in the different sections and may even be completely free of oscillation in the anode region. Similarly, Pekárek's waves of stratification<sup>11)</sup>

clearly originate in the plasma.

On the other hand, our own demonstration of anode spot oscillations without moving striations, Cooper's anode spot generated striations <sup>6)</sup>, and the detailed study by Takamine, Suga, and Yanagihara <sup>12)</sup>, show that anode spot oscillations can also give rise to moving striations.

#### D. Conclusion

We have demonstrated the dependence of moving striations upon metastable atoms in the positive column plasma of a glow discharge by experiments in alkali metal vapors, which have no metastable states and no moving striations, and by experiments in the inert gases, where the metastable population and the striation behavior can be greatly changed by the technique of irradiative depopulation. The metastable dependence seems so clearly demonstrated that it now becomes worthwhile to attempt a more polished theory of striations.



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## 2. TRANSPORT PROPERTIES IN A ROTATING PLASMA

Harry S. Robertson

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### I. A Paradox

Because of a radial electric field, a cylindrical plasma in an axial magnetic field rotates about its axis. In the absence of collisions, the guiding center drift velocity is given by  $\underline{v} = \underline{E} \times \underline{B} / B^2$ , in MKS units. Since collisions are often present, either with neutral background gas or with confining walls, there should apparently exist a frictional mechanical torque on the rotating plasma which, if not balanced by an electromagnetic torque, would suppress the plasma rotation. But from a microscopic point of view, it is evident that any non-axial motion of the charged plasma particles produces the guiding-center drift around the axis. Therefore, in the absence of purely axial streaming of a confined plasma, the presence of friction and its consequent mechanical torque would seem to require the existence of an electromagnetic torque. There is a paradox in connection with this point which will be examined in this report. A radial current allows a  $\underline{j} \times \underline{B}$  force to supply the torque, but when the radial current is zero, difficulties arise. It is suggested that when radial current is impossible, i. e., when ambipolar diffusion is required, then either the net frictional torque is zero or cooperative phenomena must be present, and the diffusion process itself

is thereby enhanced.

Denoting the mechanical and electromagnetic angular momenta of the plasma by  $\underline{L}_m$  and  $\underline{L}_e$ , respectively, and the corresponding torques by  $\underline{M}_m$  and  $\underline{M}_e$ , we have

$$\frac{d}{dt} (\underline{L}_m + \underline{L}_e) = \underline{M}_m + \underline{M}_e \quad (1)$$

For a steady state plasma, the time derivatives are zero, with the result that  $\underline{M}_e \neq 0$  if  $\underline{M}_m \neq 0$ . The electromagnetic torque may be written as <sup>(1)</sup>

$$\underline{M}_e = \int (\underline{E} \times \underline{D} + \underline{H} \times \underline{B}) dV + \int \underline{r} \times (\underline{E} \underline{D} + \underline{H} \underline{B}) \cdot d\underline{S} - \frac{1}{2} \int (\underline{E} \cdot \underline{D} + \underline{H} \cdot \underline{B}) \underline{r} \times d\underline{S} \quad (2)$$

where the first integral is over the volume of the plasma considered, and the other two integrals are over the surface enclosing that volume. In cylindrical geometry, with  $\underline{D} = \epsilon \underline{E}$  and  $\underline{B} = \mu \underline{H}$ , and with the most general choice of  $\underline{E}$  and  $\underline{H}$ , the quantities of interest may be written  $\underline{R} = \underline{m}R + \underline{k}Z$ ,  $\underline{E} = \underline{m}E_R + \underline{n}E_\theta + \underline{k}E_Z$ , and similarly for  $\underline{H}$ , where the unit vectors ( $\underline{m}$ ,  $\underline{n}$ ,  $\underline{k}$ ) form a right-handed basis. Without imposing any symmetry requirement, the Z-component of  $\underline{M}_e$ , integrated over a cylinder of

radius  $R_1$  from  $Z = Z_1$  to  $Z = Z_2$  is given by

$$M_{ez} = R_1^2 \int_{Z_1}^{Z_2} \int_0^{2\pi} (\epsilon E_R E_\theta + \mu H_R H_\theta) d\theta dz + \int_0^{R_1} \int_{Z_1}^{Z_2} (\epsilon E_z E_\theta + \mu H_z H_\theta) R^2 d\theta dR \Big|_{Z_1}^{Z_2} \quad (3)$$

where the fields in the first integral are evaluated at  $R = R_1$ , and the

notation on the second integral indicates that the term is to be evaluated as the difference between the contributions of the two ends. Note that the

first integral in Eq. (2) vanished because the integrands are zero, and the third integral has no Z-component.

For a steady state plasma in a uniform axial magnetic field, it might seem reasonable to require that  $E_\theta = 0 = H_R$ , which causes the first integral of Eq. (3) to vanish, and  $\frac{\partial H_Z}{\partial Z} = 0 = \frac{\partial H_\theta}{\partial Z}$ , which

causes the second integral to vanish also. If  $M_{eZ} \neq 0$ , the hypotheses of Z-independence and vanishing  $E_\theta$  and  $H_R$  are inconsistent.

If there is radial current, the second integral of Eq. (3) is not zero, since the axial current is not Z-independent. It is easy to show that this term reduces to a torque produced by the  $\underline{j} \times \underline{B}$  force. As shown by Anderson <sup>(2)</sup> et al., the radial current density is proportional to the coefficient of viscosity, and the electromagnetic torque from  $\underline{j} \times \underline{B}$  can be shown to be just sufficient to balance the viscous torque. If the viscosity were zero in this analysis, there would be no need for radial current. Diffusion has been neglected, and the treatment of viscosity is the simplest possible, but their results illustrate clearly that the viscous drag causes the required electromagnetic torque.

The case of particular interest here is that for which the second integral of Eq. (3) vanishes because of Z-independence. In this case, Eq. (3) may be rewritten to give

$$M_{eZ} = R_1^2 \Delta Z \int_0^{2\pi} (\epsilon E_R E_\theta + \mu H_R H_\theta) d\theta, \quad (4)$$

where  $\Delta Z = Z_1 - Z_2$ . Obviously  $M_{eZ}$  here is zero for steady state,

symmetric fields, and if  $M_{eZ}$  is zero, the  $M_{mZ}$  must also be zero.

This is the paradox, that there can exist a steady rotation in the presence of friction without any loss of angular momentum or need for torque. A detailed analysis of this point is given in the next section of this report.

The time average of the Z-component of torque, denoted by

$\langle M_{eZ} \rangle$  is given by

$$\langle M_{eZ} \rangle = \lim_{\tau \rightarrow \infty} \frac{R_1^2 \Delta z}{\tau} \int_0^\tau \int_0^{2\pi} (\epsilon E_R E_\theta + \mu H_R H_\theta) d\theta dt \quad (5)$$

An interchange of the order of integration, and the symmetry argument that the time average should be independent of  $\theta$ , gives

$$\langle M_{eZ} \rangle = 2\pi R_1^2 (\epsilon \langle E_R E_\theta \rangle + \mu \langle H_R H_\theta \rangle) \quad (6)$$

Random fluctuations in current and charge densities lead to zero for each term of Eq. (6), in contradiction to the expectation that

$\langle M_{eZ} \rangle \neq 0$ . The plasma fluctuations required, then, appear to be not entirely random, but otherwise they may be quite like ion oscillations evoked by Spitzer<sup>(3)</sup> to account for enhanced ambipolar diffusion losses in the Stellerator during ohmic heating. There appears to be sufficiently good coupling between currents confined to neighboring tubes of force to invalidate any assumption that ion waves in these neighboring tubes are completely independent.

The excitation of correlated fluctuations, or cooperative oscillations, in a plasma as a consequence of random collisions with walls or

background gas may seem implausible at first. It is perhaps no more implausible than the excitation of highly ordered motion in a violin string in response to the frictional drag of the bow. The system responds in its characteristic, but not necessarily invariant way to a variety of forcings.

## II. Steady State Theory

The problem of torque on a cylindrical rotating plasma in an axial magnetic field corresponds to that of the  $y$ -force on a plasma transported principally in the  $x$ -direction across a magnetic field  $B_z$ . If there is an applied electric field  $E_x$  and a concentration gradient in the  $x$ -direction, there will be both  $x$ - and  $y$ - particle flows. In a steady state system, the relationship between frictional and electromagnetic forces will be established.

Suppose that the electric field in the  $y$ -direction is zero, and that all the field quantities are taken as constant in time.

In an isotropic fluid, the relationship defining the mobility  $\mu$  is  $\underline{U} = \pm \mu \underline{E}$ , where the sign choice is usually that of the electric charge of the drifting particle. The basic definition of  $\mu$  is to be retained for this analysis, with the generalization that  $\mu$  may be a function of  $\underline{E}$ . It is customary to represent  $\mu$  in a magnetic field as a tensor of second rank, with  $\underline{E}$  representing the actual electric field. An entirely equivalent representation, which diagonalizes the mobility

tensor, is to write  $\underline{E}$  as  $\underline{E}'$ , the field as seen by particles drifting with the average drift velocity. This alternative formulation makes clear that the reason mobilities and diffusivities are modified in the magnetic field is that the drifting particles see an electric field which differs from that as seen in the laboratory.

The average drift velocities of ions across the magnetic field for the situation described are given by

$$U_x = -D \frac{\partial}{\partial x} \ln N + \mu_x (E'_x) E'_x, \quad (1)$$

$$\text{and } U_y = \mu_y (E'_y) E'_y, \quad \text{where} \quad (2)$$

$N$  is the ion concentration, assumed independent of  $y$ ,  $\mu_x (E'_x)$  and  $\mu_y (E'_y)$  are the  $x$  - and  $y$  - mobilities, not necessarily equal, and indicated as possible functions of the fields as seen by the drifting ions, which are given by

$$E'_x = E_x + U_y B_z, \quad (3)$$

$$\text{and } E'_y = -U_x B_z \quad (4)$$

Substitution of  $E'_y$  from Eq. (4) into Eq. (2) gives

$$U_y = -\mu_y U_x B_z, \quad (5)$$

where the indicated functional dependence of  $\mu_y$  has not been written out.

Substitution of Eq. (5) into Eq. (3) gives

$$E'_x = E_x - \mu_y U_x B_z^2, \quad (6)$$

and substitution of this result into Eq. (1) gives

$$U_x = -D \frac{\partial}{\partial x} \ln N + \mu_x E_x - \mu_x \mu_y U_x B_z^2 \quad (7)$$

Eq. (7), when solved for  $U_x$  gives finally

$$U_x = \frac{-D}{1 + \mu_x \mu_y B_z^2} \frac{\partial}{\partial x} \ln N + \frac{\mu_x E_x}{1 + \mu_x \mu_y B_z^2} \quad (8)$$

Thus the diffusivity and mobility are seen to be reduced by the factor  $1/(1 + \mu_x \mu_y B_z^2)$ , which is equivalent to the well-known factor  $1/(1 + \omega_c^2 \tau^2)$  when the mobilities are equal and given by  $e\tau/m$ ,  $\omega_c = eB/m$ , the cyclotron frequency, and  $\tau$  is the relaxation time. The mobilities differ in the x- and y- directions if the relaxation times differ, and the reduction factor may then be written as  $1/(1 + \omega_c^2 \tau_x \tau_y)$ . (The mobilities, or relaxation times may differ, for example, in a two component partially ionized plasma, in which ions and electrons are drifting together in the x- direction because of a concentration gradient, but are drifting oppositely in the y- direction because their charges are opposite. Each species then must flow upstream with respect to the other species and downstream with respect to its own species in the y- direction, and downstream for both species in the x- direction. Thus mobilities may well be different in the two directions.)

In order for a steady state mobility to exist, it is necessary that the frictional force be equal and opposite to the electromagnetic force. For a particle drifting with constant average velocity in the y- direction, for example, the sum of the electrical and frictional (viscous) forces must be zero, or

$$eE_y + F_y = 0 \quad , \quad (9)$$



where  $F_y$  is the frictional force and  $E_y$  is again the electric field as seen by the drifting particle. From Eq. (4), this gives

$$F_y = eU_x B_z \quad (10)$$

and the net frictional force per unit volume on the positive ions is

$$N F_y = eNU_x B_z = j_{ix} B_z \quad (11)$$

where  $j_{ix}$  is the x - component of the ion contribution to the electric current density. There is, of course, an equal and opposite force exerted by the drifting ions on the medium through which they drift.

An exactly similar analysis can be made for the electron component, giving the equivalent of Eq. (11) with  $j_{ex}$  in place of  $j_{ix}$ . The net force density, then, on a drifting plasma is

$$(F_y)_{\text{net}} = (j_{ix} + j_{ex}) B_z = j_x B_z \quad (12)$$

where  $j_x$  is the current density.

This is not surprising except in the case of  $j_x = 0$ . In ambipolar diffusion, for example, the radial current density is zero, there being equal positive and negative contributions to it. The unexpected result is that, despite differences in x- and y- mobilities and in ion and electron mobilities, the net force in the y- direction on the diffusing plasma is just that given by  $j_x B_z$ , even when  $j_x$  is zero.

The net momentum density in the y direction is given by

$$\begin{aligned} eP_y &= m_i j_{iy} + m_e j_{ey} \\ &= -m_i \mu_{iy} j_{ix} B_z - m_e \mu_{ey} j_{ex} B_z \end{aligned} \quad (13)$$

When  $j_{ex} = -j_{ix}$ , as in ambipolar diffusion, Eq. (13) gives

$$eP_y = j_{ix} B_Z (m_e \mu_{ey} - m_i \mu_{iy}). \quad (14)$$

In the usual relaxation-time approximation,

$\mu_{ey} = e\tilde{\tau}/m_e$  and  $\mu_{iy} = e\tilde{\tau}_i/m_i$ , so Eq. (14) becomes

$$P_y = j_{ix} B_Z (\tilde{\tau}_e - \tilde{\tau}_i). \quad (15)$$

Since  $\tilde{\tau}_i$  is probably much greater than  $\tilde{\tau}_e$ , it is probable that  $P_y$  is in the negative  $y$ -direction for positive  $j_{ix}$  and  $B_Z$ . This is the direction in which the ions are deflected, as might be expected.

It is thus seen that in steady state transport, there is a non-zero momentum density perpendicular to the magnetic field and to the expected direction of transport across the field. In cylindrical geometry, the radial transport produces a non-zero axial angular momentum density with zero net torque.

### III. Conclusion

If classical diffusion theory is applicable to the physical situation in a plasma, it is possible for rotational motion to exist steadily in the presence of friction. No analysis has been made of the stability of the steady state solution to small perturbations, and it is possible that such an analysis might show that conditions may exist for which the steady state is not stable.

Classical diffusion theory seems to become less meaningful, however, as the plasma becomes more nearly fully ionized. Relaxation times become longer, cooperative interactions more probable, and the

possibility of the correlated terms in Sec. I which would lead to non-zero electromagnetic torques is not remote. It thus seems that there may be the possibility of frictional forcing of plasma cooperative modes, with enhanced diffusion as a consequence.

Spitzer's calculation <sup>(3)</sup> of diffusion, based on ion waves propagating along Z, produces correlated  $E_R E_\theta$  as seen by an ion lying on an adjacent line of force, but his assumption that the ion waves travelling on different lines of force are uncorrelated would seem to lead to zero net torque for any symmetric average ion distribution. His uncorrelated ion waves lead to diffusion in the random walk sense, as he has shown. It seems, however, that the ion waves should not be uncorrelated, since ions gyrate in rather large orbits, communicating very effectively with other ions whose guiding centers lie on adjacent lines of force, and the electric fields are also effective in correlating ion motion. There is need for further experimental and theoretical work on this point, particularly since a large scale correlated motion would apparently produce a high diffusion rate and may arise from frictional forcing.

Neidigh and Weaver <sup>(4)</sup> have described a rotating plasma system with just the kind of correlations required in Sec. I. We have observed in our laboratory, in a wide variety of rotating plasma systems, quite high amplitude, almost periodic oscillations, which may well be related to the transport phenomena discussed in this report. A detailed survey

of the experimental results will be reported later.

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### III. ARC STUDIES IN A MAGNETIC FIELD

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#### A. Introduction

These studies concern the phenomena known as the Neidigh pressure gradient arc, or Mode II, which was first observed by Neidigh and Weaver <sup>(1)</sup>. The arc was observed when electrons from a heated filament were accelerated by an applied electric field into a region into which gas was being introduced. The arc components were within a vacuum system and an axial magnetic field was imposed. A schematic diagram of Neidigh and Weaver's apparatus is shown in Figure 1. The two end plates were made of 1/2" thick copper plates approximately 6" square. The circular holes in the end plates through which the arc passed were approximately 1/4" in diameter and a tube fed gas from an external supply into the hole in the end plate near the filament. Electrically the end plates were at the same potential as the metal vacuum system. The filament was maintained at potentials from 0 to -500 v with respect to the end plates.

Shielded probe measurements indicated time varying potentials in the region around the arc with peak to peak voltages on the order of the applied arc voltage. The frequencies observed varied from 20,000 cps to 100,000 cps depending on system parameters. Sputtering patterns

Figure 1

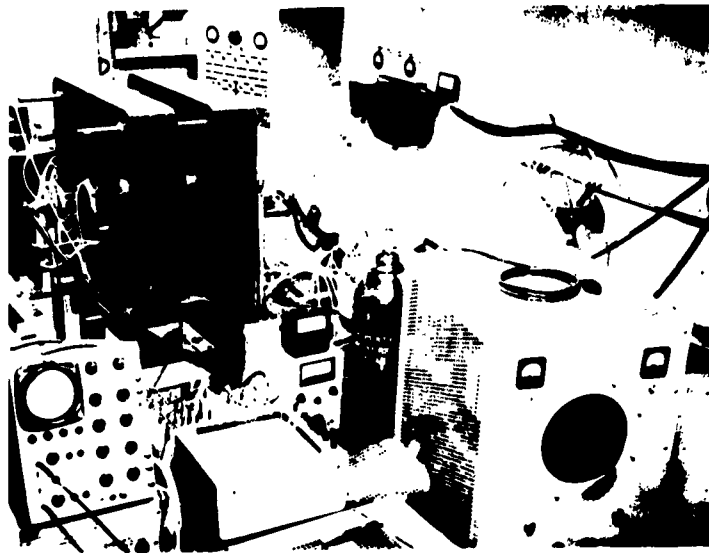
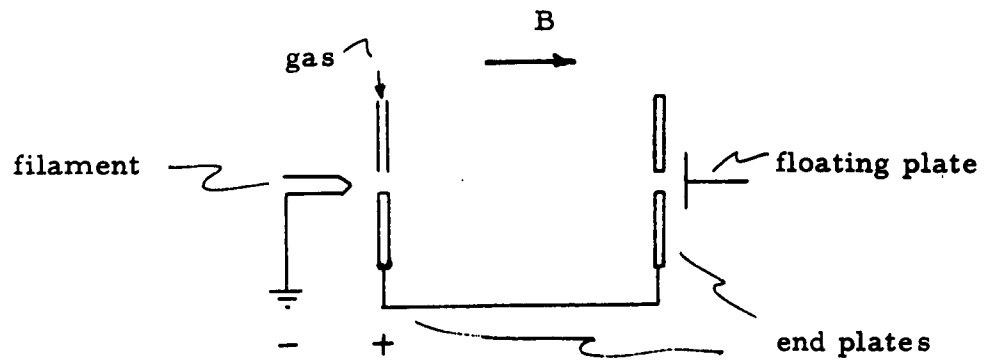


Figure 2

and the radius of the observable glow indicated that high energy ions were present in the arc.

Later measurements using a crossed field velocity analyzer <sup>(2)</sup> indicated that the ions possess energies several times higher than would be expected from the applied arc voltage. This investigation also showed that the ion energies apparently do not depend on the applied magnetic field.

It is important to remember in considering this arc that the phenomena observed are taking place in a region which is free of externally imposed electric fields and that the electric field applied between the filament and one end plate is in a direction opposite to the motion of the ions.

#### B. Experimental Facility.

The arc facility which has been constructed at the University of Miami is now in operation. Its design was based on the experience of Neidigh and Weaver at ORNL.

The vacuum system is constructed of pyrex glass components 3" and 6" in diameter. The system is connected to two 6" oil diffusion pumps backed up by Welch model 1640 mechanical pumps; each pump station is equipped with a closed cycle Freon 12 cooled baffle and a valve so that each or both pumps can be shut off from the vacuum system. With this arrangement, the pumping speed of the system can be varied (e.g.) from 0 to 1,000 liters/sec. at  $10^{-4}$  mm Hg. Additional provision

has been made for a liquid nitrogen cooled trap within the system. The pressures at various positions within the system are measured with ion gauges and thermocouple gauges. The control panel for the vacuum system has provision, if desired, for automatic shutdown in the event of power failure or system pressure loss during the night.

The section of the vacuum system in the magnetic field consists of a cross arrangement with 6" diameter along the field direction for the arc and 3" at right angles to the field to provide access for probes.

The magnetic field is supplied by a set of Helmholtz coils with an inner diameter 14" and a separation between coils of 5" when set for maximum field uniformity. The nominal radius of the coils is 11.5". Power for the magnet coils is supplied by 3 Miller SRH 444 D.C. power supplies capable of providing 400 amps at 120 volts D.C. This arrangement is capable of producing fields from 0 to 3,300 gauss. In operation a bank of 875 amp, hr. batteries can be connected in parallel with the power supplies to reduce ripple. The magnetic field is measured with a Radio Frequency Laboratories Model 1890 gauss meter. This gauss-meter operates on the Hall Effect principle. The power supplies are controlled remotely so that the magnetic field can be conveniently adjusted at the control station.

The filament is made from 1/8" diam. tantalum rod. The filament supply is a D.C. "K" type supply capable of providing over 400 amps filament current. The "K" supply has an unfiltered ripple of as much



as 25 percent. The effect of this comparatively low frequency ripple was negligible with smaller arc geometry but was most pronounced in its effect on the most recent electrode arrangement. To avoid this difficulty, the filament is now heated with current from 6 (six) 875 ampere hour batteries in parallel with the "K" supply. This arrangement permits variation of filament current with minimum ripple (0 - 5 percent) since the batteries provide most of the filament power.

The arc voltage is provided by a 0-600 volt 0-32 amp D. C. current and voltage controlled supply.

Power for all auxiliary equipment such as oscilloscopes, recorders and meters is provided through a 5 kw Sorensen voltage regulator.

A gas introduction manifold permits the control and measurement of the input of gas into the vacuum system. H, N, He and A are presently available.

Electrodes similar to the Oak Ridge arrangement have been constructed within the vacuum system. Measurements are in progress to determine overall arc characteristics.

A photograph of the research facility is shown in figure 2.

### C. Experimental Results.

The first characteristics measured were "diode" characteristics that is  $I_a = f(V_a)$  where  $I_a$  is the arc current flowing between the cathode and the end plates and  $V_a$  is the arc voltage between the cathode and the

end plates.  $I_a = f(V_a)$  was measured with and without gas flow and with and without magnetic field. Typical results are shown in figure 3.

The most striking feature of these observations was that with gas flow and magnetic field a distinct hysteresis occurs in the  $I_a = f(V_a)$  curve.

It was observed that oscillations were initiated when the discontinuity in  $I_a = f(V_a)$  first occurs and extinguished when the hysteresis loop is completed as shown in figure 4. The oscillations were detected by a capacitance probe around the outside of the glass wall of the system.

The discontinuity in  $I_a = f(V_a)$  was always accompanied by marked changes in the visual appearance of the arc. A typical photo of the oscillations as observed with an oscilloscope are shown in figure 5.

Oscillations were also observed simultaneously in the voltage appearing on the floating plate. All the evidence thus far indicates that the upper part of the hysteresis curve corresponds to the Mode II phenomena observed by Neidigh and Weaver.

Some thought was given to the possibility that an analog using electron tubes might shed light on the type of mechanisms involved. An analog was found and is illustrated in figure 6. This work served mainly to indicate a similarity to the behavior of glow discharge tube characteristics as observed by Robertson and Hakeem. A cesium vapor glow discharge tube operating in a magnetic field had exhibited indications of a hysteresis loop in its  $I = f(V)$  characteristics. The possibility was too much to ignore. Using an x - y recorder, character-

Figure 3

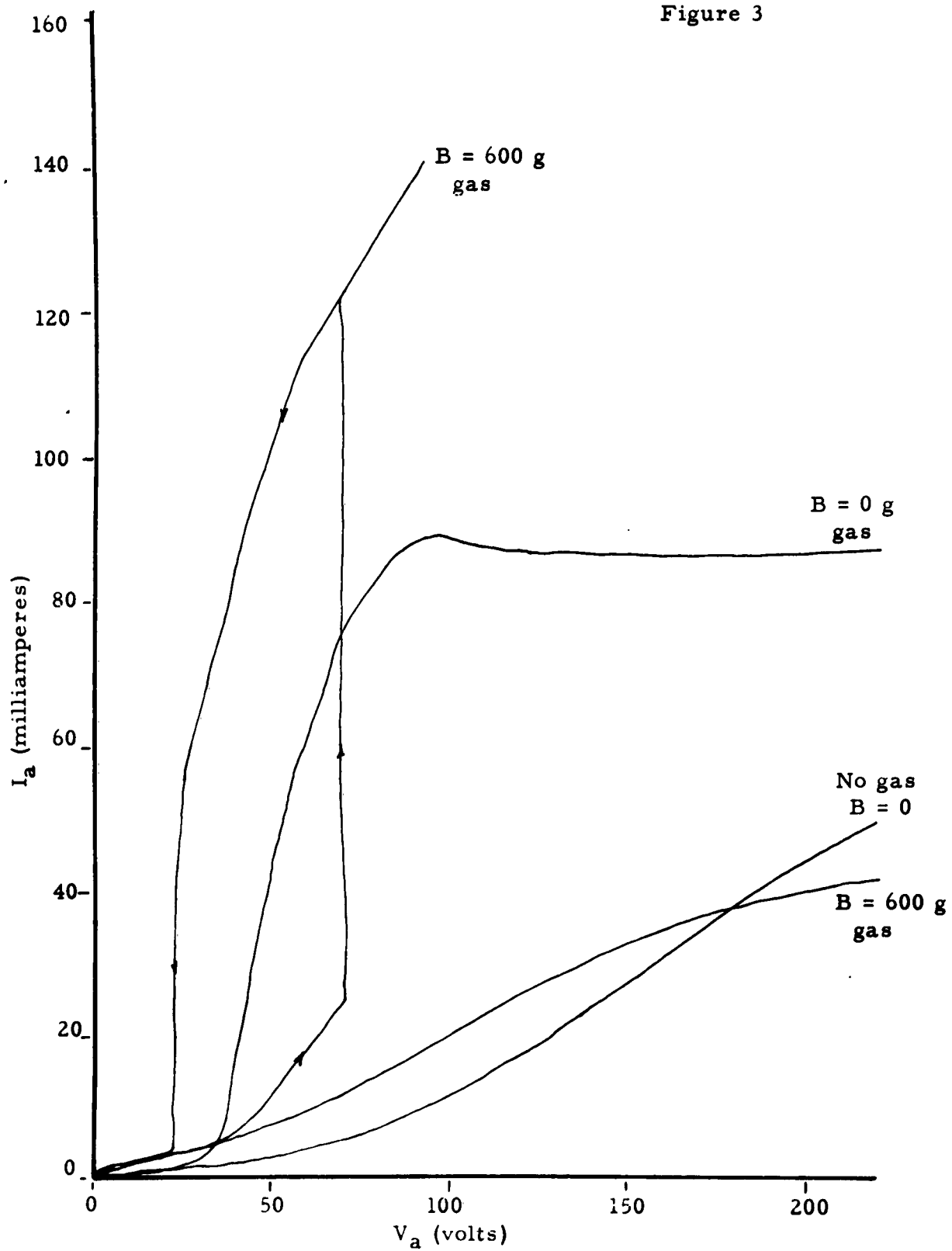
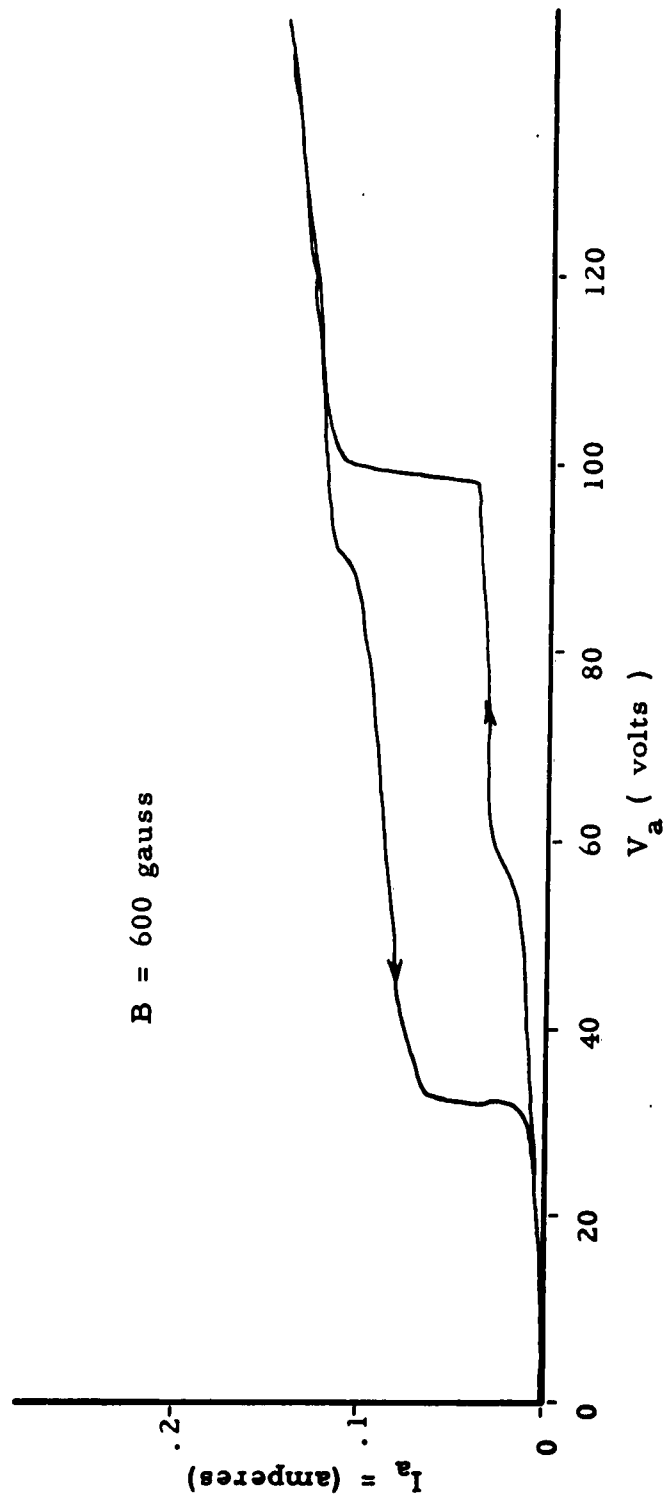


Figure 4



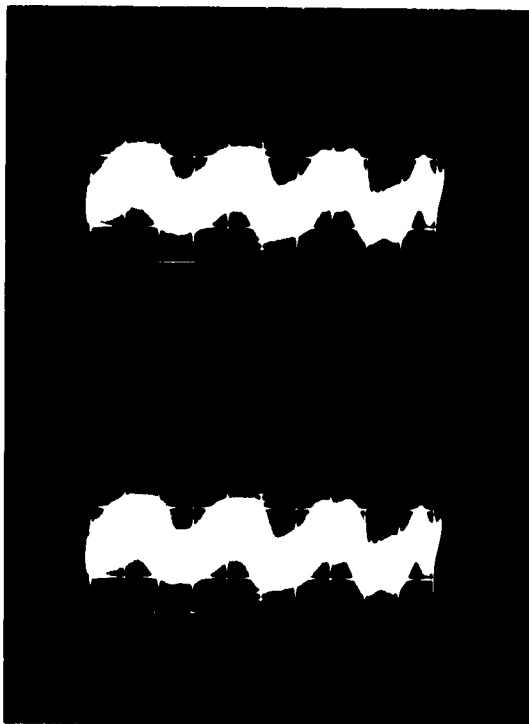


Figure 5

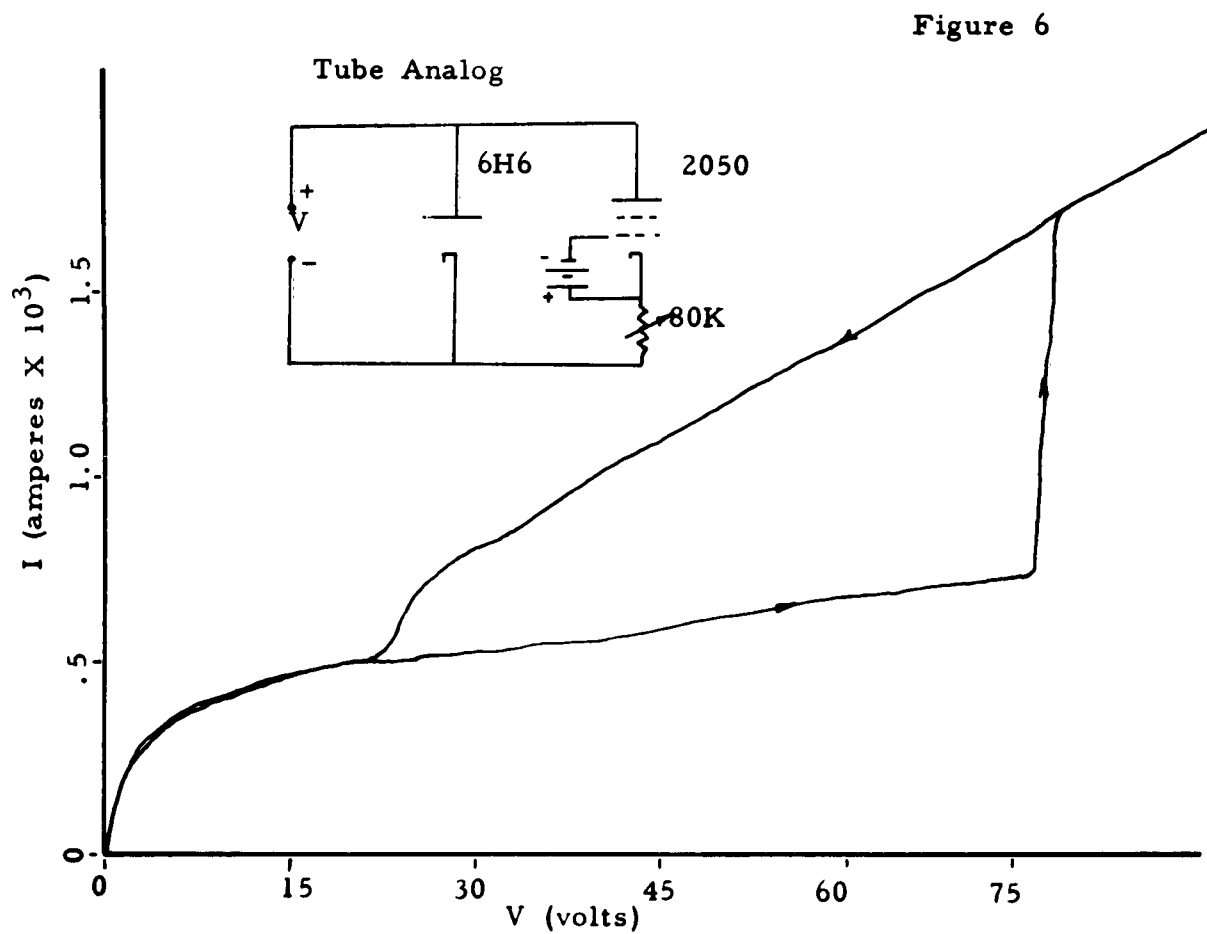


Figure 6

istic curves for the glow discharge tube were run. The hysteresis loop was always present when effects such as described by Lehnert (3, 4) were observed and as suspected oscillations in the tube voltage were observed to occur when the tube was operated in the upper branch of the hysteresis loop. The jocular observation made at the time, that we had seen Mode II in a cesium glow discharge, may prove to be extremely meaningful. It can be definitely stated that a connection exists between the "enhanced diffusion" observed by Lehnert and the "Mode II" observations of Neidigh and Weaver.

The "enhanced diffusion" regime observed by Lehnert by increasing the magnetic field can be induced by varying other discharge parameters. The "Mode II" regime which Neidigh and Weaver observed by reducing background pressure can also be induced by varying other arc parameters. The transition into "Mode II" depends on magnetic field in a manner similar to the "enhanced diffusion" effects. This can be seen in figures 7 and 8. Figure 7 shows the arc characteristic  $I_a = f(V_a)$  for various values of magnetic field. There was almost no dependence on magnetic field below 1,500 gauss and above 1,900 gauss but a marked change occurs between these values. The high current operation corresponds to "Mode II". Figure 8 shows  $I_a = f(B)$  where the arc voltage is held constant at 200 volts. As the magnetic field increases, a sharp discontinuity occurs which corresponds to the transition from "Mode I" to "Mode II". When the magnetic field

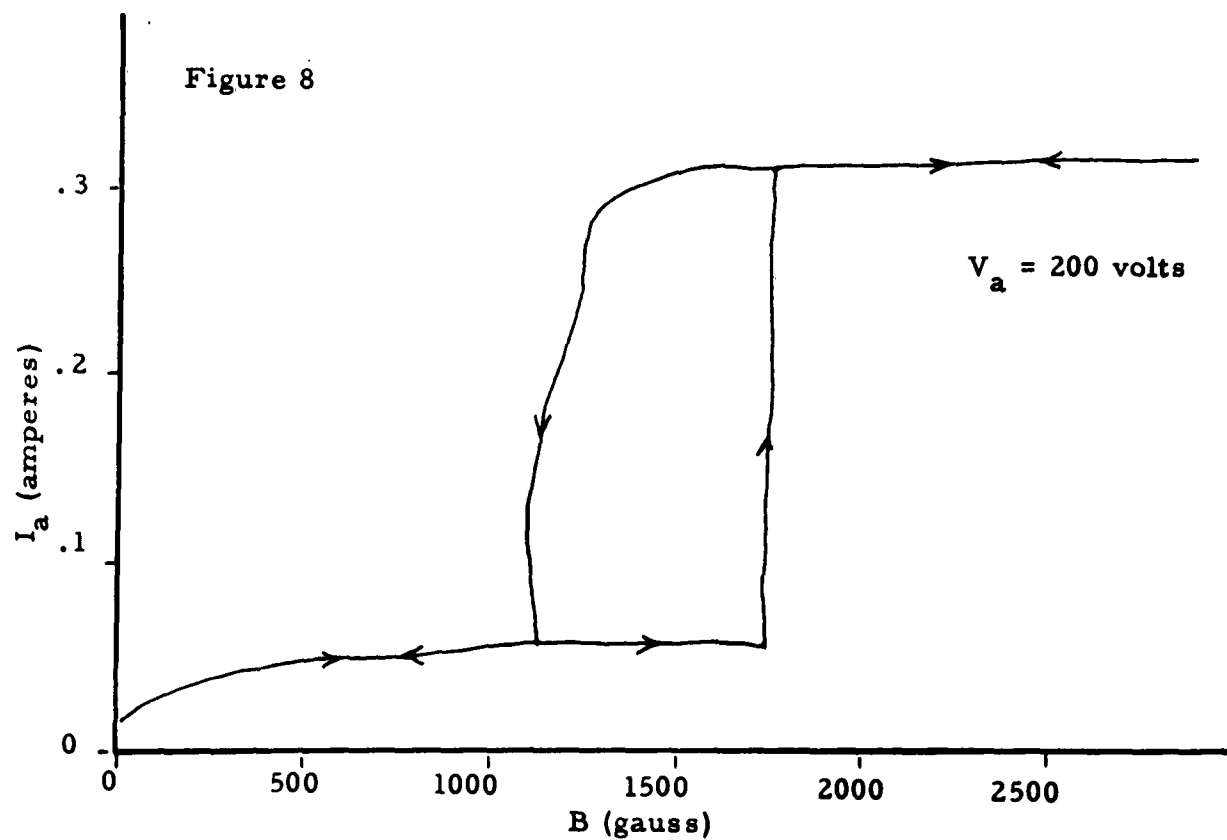
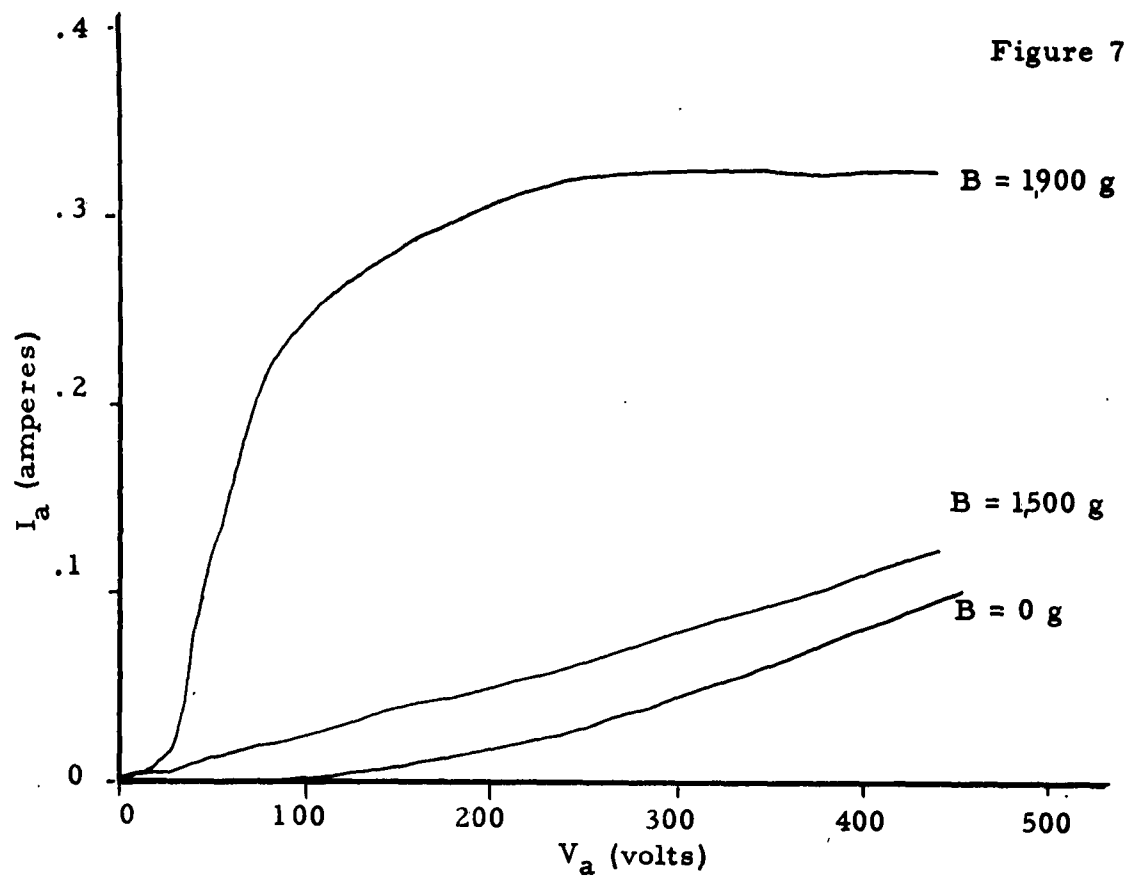
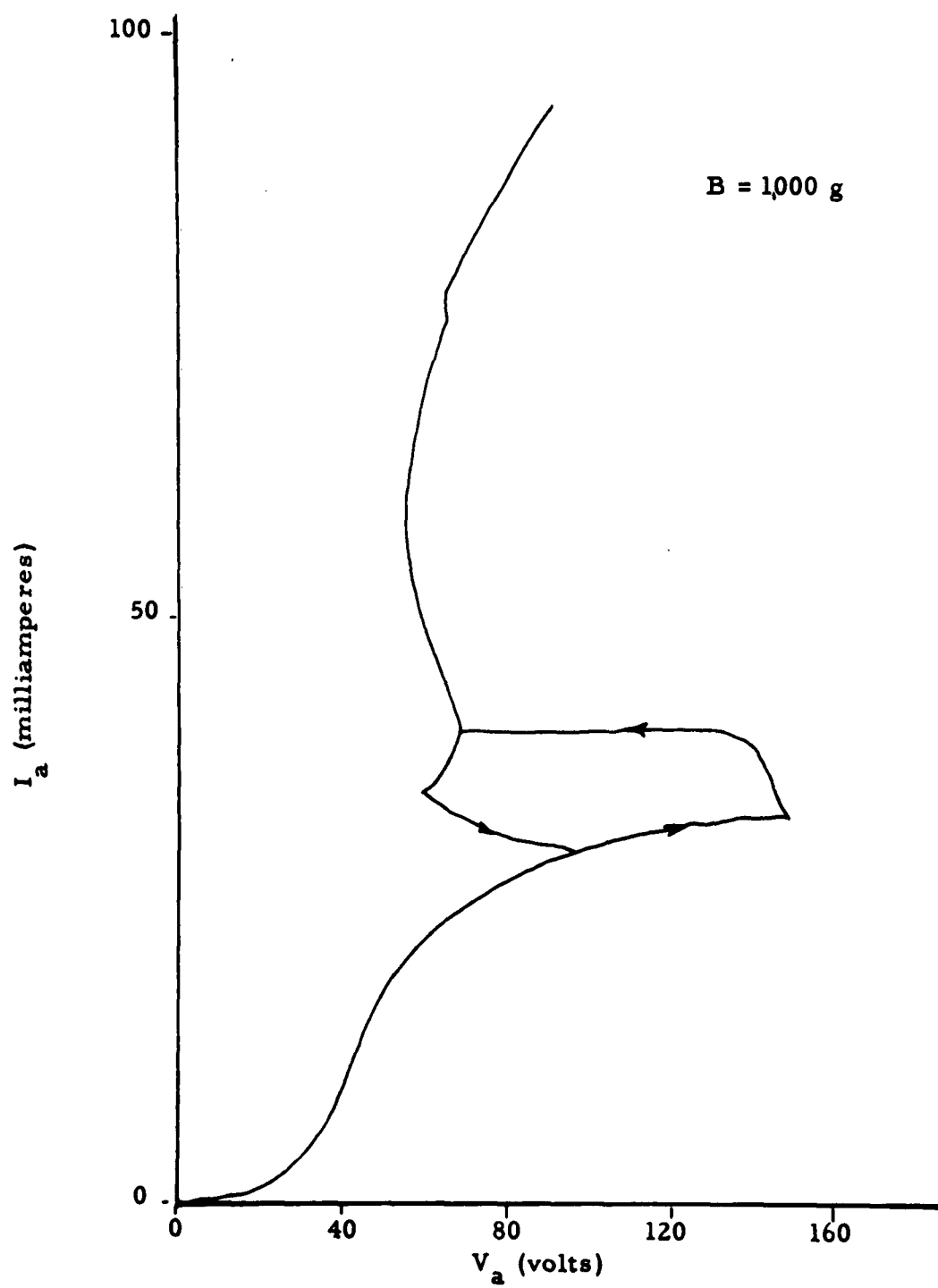


Figure 9





is reduced the curve does not retrace. A distinct hysteresis is always observed. It should be noted that observations made by short time measurements at discrete values of magnetic field will result in only one branch being detected.

There appears to be a growing awareness of the possible connection between "enhanced diffusion" and losses in thermonuclear devices. Much effort is being expended to verify the model proposed by Kadomtsev et al.<sup>(5)</sup> Our observations indicate the necessity of investigating the relationship between "enhanced diffusion" and "Mode II" further. It would seem that if Kadomtsev's theory is to be valid it should describe "Mode II" as well. Although ions are transported across the magnetic field and the rate of transport seems to depend on the magnetic field, the use of the term "enhanced diffusion" to describe this loss mechanism may be both inappropriate and misleading.

The oscillations observed to accompany "Mode II" must have some driving force which we do not as yet understand. Analogy with vacuum tube circuitry led Brodzinsky<sup>(6)</sup> to suggest that running characteristic curves with a constant current source might reveal interesting features of the hysteresis loops. The results of such an experiment are shown in figure 9. The current is a multi-valued function of the voltage and regions of distinct "negative resistance" are observed.

Now that the arc components are assembled in the 6 inch by

3 inch cross vacuum system, access for probes is available. Insulated wire probes and a cross-field analyzer probe are installed with provision for adjusting their position with respect to the arc from outside the vacuum system. Future work utilizing these probes will help to clarify the behavior of the ions.

Most of the work described above has been incorporated into the thesis submitted by Jack L. Tunstall in partial fulfillment of the requirements for the degree of Master of Science from the University of Miami.

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